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ON THE SOCIOBIOLOGY OF HAWKS AND DOVES
A phase diagram for integration and segregation

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Abstract

Competing groups in a population will be integrated or segregated depending on their contest strategies.

In this work a population of a fixed proportion of hawks and doves is supposed to be able to employ two different contest strategies, one more competitive than the other one. Energies are derived for populations employing these strategies and these energies depend on the availability of the resource for which hawks and doves compete.

The energy for the less competitive strategy is lower than the other one when the resource is abundant. In that case hawks and doves can be in cohabitation in all proportions. If, however, the resource is scarce, the energy of the more competitive strategy is lower than the other one. In that case complete segregation of hawks and doves into colonies will result.

The situation is akin to the phase pressure diagram of a binary solution with eutectic point, miscibility gap in the liquid phase and complete miscibility in the vapour phase.

Keywords: game theory, integration and segregation, phase diagrams

Introduction

A population is considered consisting of a single species but with two characters, hawks and doves, in a fixed proportion, who compete for a resource with possibly two sets of rules. If the hawk-dove population employs one set of contest rules, – or strategy –, it is said to be in one phase, if it employs the other set of rules it is in the other phase.

The situation is akin to the equilibrium of a binary solution which has two phases, liquid and vapour (say). And that similarity has motivated this paper.

The principal ingredient is the knowledge of population energies in each phase. The form of these energies can be derived from the rate laws of sociobiology that lead to evolutionarily stable strategies. In the present case both energies depend upon the availability of the resource in such a manner that, if the resource is abundant, the energy corresponding to the less competitive strategy is lower than the other one. Thus the population assumes the less competitive strategy, which will turn out to mean cohabitation of hawks and doves. When the scarcity of the resource grows, the energies

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exchange their roles. The energy corresponding to the more competitive strategy becomes lower than the other one. The population therefore assumes the more competitive strategy and this will be seen to lead to a complete segregation into colonies of hawks and colonies of doves.

When the energies of the phases are comparable, – at intermediate values of the resource –, we shall be able to define regions in which only partial segregation occurs. Growing scarcity of the resource tends to inhibit cohabitation of hawks and doves, it promotes segregation into colonies.

Population energies

Energies of two phases

We consider a population with only two characters, hawks and doves, who are capable of competing for a resource with two strategies. The value of the resource is variable, because it depends on the availability, and we shall denote it by 50 V. The value *V* is high, when the resource is scarce and it is low when the resource is reasonably abundant. The contest strategies are called **A**, – for the anxious or timid behaviour of the doves –, and **B**, – for the bold behaviour of the doves. These strategies may succinctly be described as follows.*

A: A hawk fights for the resource; a dove merely engages in a symbolic conflict, posturing and threatening but not actually fighting. Between two hawks there will always be a fight until one is injured. The winner will get the resource worth 50 V points, while the loser will get –100 points for being injured. Over many fights each hawk will thus 'win' ½ (50 V–100) points per fight. If a hawk meets a dove, the hawk will always win the resource and there will be no injury. If two doves meet, they will spend a long time posturing. One will eventually win the resource, but they will both get –10 points for wasted time: In the mean an encounter will thus gain a dove ½(50 V–20) points.

B: A hawk still fights for the sole possession of the resource, again valid 50 V points. But the doves will not timidly stand by to let the hawks have their way; they will try to steal the resource from the hawks and we assume that four out of ten times they are successful. But, successful or not, they risk injury with a penalty of –100 points. Therefore a hawk earns ½ (50 V–100) points in the mean, if it meets another hawk and 1/10·6·50 V, if it meets a dove. The contest between doves again consists of posturing and threatening with the same reward and penalty as before. Thus a dove–dove encounter will bring ½(50 V–20) in the mean and a dove-hawk encounter will average 1/10·4·50 V–100 for the dove.

We shall say that the population is in 'phase A', if it has the strategy **A** and in 'phase B', if it has strategy **B**. The more striking difference is in the doves between

* People familiar with the sociobiological literature will recognize strategy A as the one proposed by J. Maynard Smith and G. R. Price [1]; the only difference here is the variable value *V*. In [1] *V* was equal to 1. The wording of strategy A is close to a verbatim quote from P. D. Straffin [2]

phases A and B, because they behave boldly rather than timidly. But also the hawks behave differently, albeit by reaction: While in phase A they leave the doves alone, they are ready to hurt them in phase B. Let x_H and $x_D=1-x_H$ be the fractions of hawks and doves in the population. The expectation values E_H and E_D for the number of points won by hawks and doves respectively are

$$\begin{aligned}\textbf{A:}\ & E_H=(25\text{ V}-50)\,x_H+50\text{ V}\,x_D\\ & E_D=0\,x_H+(25\text{ V}-10)\,x_D\\ \textbf{B:}\ & E_H=(25\text{ V}-50)\,x_H+30\text{ V}\,x_D\\ & E_D=(20\text{ V}-100)\,x_H+(25\text{ V}-10)\,x_D\end{aligned} \tag{1}$$

since x_H and x_D are the probabilities that an individual meets hawk or dove.

In sociobiology it is assumed that a difference in expectation values translates into an evolutionary payoff for a character. Thus if $E_H>E_D$ holds we expect the hawk fraction x_H to grow in time. We write**

$$\textbf{A:}\ \frac{dx}{dt}=\alpha(E_H-E_D) \qquad (\alpha>0) \tag{2}$$

For the two phases of the population this means

$$\begin{aligned}\textbf{A:}\ & \frac{dx}{dt}=\alpha(-60x+[25\text{ V}+10])\\ \textbf{B:}\ & \frac{dx}{dt}=\alpha(40x+[5\text{ V}+10])\end{aligned} \tag{3}$$

We may rewrite these equations in the forms

$$\begin{aligned}\textbf{A:}\ & \frac{dx}{dt}=-\alpha\frac{d}{dx}\left\{30\left(x-\frac{[25\text{ V}+10]}{60}\right)^2\right\}\\ \textbf{B:}\ & \frac{dx}{dt}=-\alpha\frac{d}{dx}\left\{-20\left(x-\frac{[5\text{ V}+10]}{40}\right)^2\right\}\end{aligned} \tag{4}$$

This version of the Eqs (3) suggests that the rate of change of x is determined by the gradient of parabolic energy functions, namely

$$F^A=30\left(x-\frac{[25\text{ V}+10]}{60}\right)^2+C^A, \qquad F^B=-20\left(x-\frac{[5\text{ V}+10]}{40}\right)^2+C^B \tag{5}$$

C^A and C^B are constants of integration. Their values are not important when the population stays in one phase or the other. This is the case which we discuss first, but only

** From here on we denote x_H by x and x_D by $1-x$

briefly, because it is frequently considered in much detail in sociobiology (*e.g.* [3]) and game theory [2].

Evolutionarily stable strategies

Here we consider one phase only, either A or B and we focus attention on the rate laws (3), or (4), and the energies (5). According to these relations the rate of change of x comes to a halt when the energy assumes a minimum. This may occur somewhere in the interval $0<x<1$, in which case there exists a mixed evolutionarily stable strategy in the jargon of sociobiology, i.e. a strategy which allows for the cohabitation of hawks and doves. The minimum of energy may also occur at the end-points $x=0$ or $x=1$, in which case there is an evolutionarily stable strategy but it excludes either hawks or doves.

Thus for $V=1$ in $(5)_1$ F^A has a minimum at $x=7/12$ so that hawks and doves may coexist with a slight preponderance of hawks. But, again for $V=1$ in $(5)_2$ the minimum of F^B is at $x=1$, so that in this case the hawkish character is evolutionarily stable.

But evolution is slow and, if a population starts out with a particular value of x, it may take a long time, – many generations – before the hawk fraction assumes the value dictated by the evolutionarily stable strategy, i.e. the energetic minimum.

Therefore we proceed to discuss the case when x is constant in a population and arbitrary, either because the population was newly created or because of a recent immigration of hawks (say).

Energies of phases

For anything interesting to occur in the case of a fixed hawk fraction x we must have a population which is capable of assuming two phases, like the phases A and B described before with different strategies and with energies that depend on the availability of the resource, and hence on its value V.

The availability of the necessary resource may change much more rapidly than the hawk fraction is altered by evolution. Indeed, during a single generation – when we may consider x to be constant – the value V may vary considerably. We proceed to investigate the effect of such a variation on the population.

The effect of a change of V on the population is determined by the energies F^A and F^B, because – roughly speaking – we expect the population to establish the phase with the lower energy. We shall be more specific a little later.

On the other hand, we expect the phase A, with its timid doves to prevail when the food supply is abundant so that V is small. This means that F^A should be smaller than F^B for low values of V and we make sure of this by choosing the constants C^A and C^B appropriately, viz.

$$C^A=0, \quad C^B=50 \tag{6}$$

Thus we can now plot the energies F^A and F^B with V as a parameter from (5) and (6). Fig. 1 shows these curves for some values V between $V=1$ and $V=3.5$. The convex curves represent F^A, while the concave ones represent F^B.

In passing we note that among the curves drawn in Fig. 1 the one for V=1.2 is the only one with a minimum in $0<x<1$. Already for V=2 the minimum has shifted to x=1. This means that the mixed evolutionarily stable strategy A is very sensitive to the availability of the resource. Even a modest scarcity of the resource destroys it and makes ‘pure hawk’ the only stable strategy.

More important for the present argument is that for V=2.5 the two curves $F^A(x)$ and $F^B(x)$ begin to intersect each other. Such intersections continue to occur until about the value V=3 whose functions $F^A(x)$ and $F^B(x)$ just barely touch. For V=2.75 there are clear intersections in two points, at about x~0.19 and x~0.92.

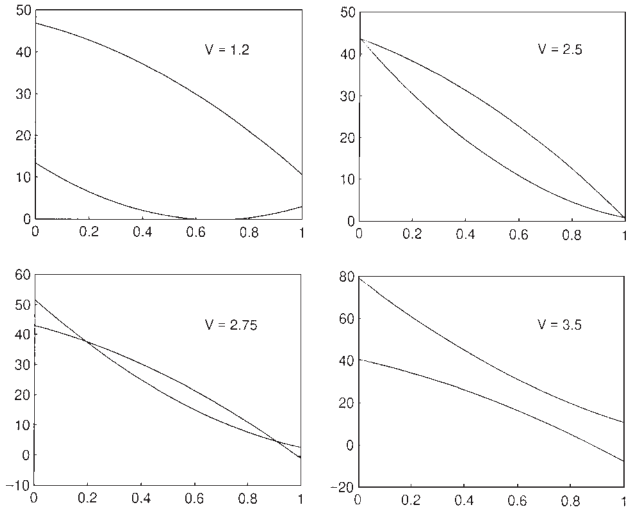

Fig. 1 Energies of the population in phase A (convex) and phase B (concave) for different values V

Phase equilibria

Phase separation, formation of colonies

We proceed on the assumption that a population for a given – and constant – hawk fraction x will assume the strategy A or B depending on the respective energies. Thus it might seem reasonable to assume that, if $F^A<F^B$ holds, the population will be in phase A, while for $F^B<F^A$ it will be in phase B.

Thus from inspection of the bold lines of Fig. 1 it would appear that, for V=2.75, the population is in phase B if $x\lesssim 0.19$ holds and again for $x\gtrsim 0.92$. In between it should be in phase A and in all cases there might be cohabitation of hawks and doves.

However, nature has a more interesting alternative. At least the inanimate solution of two fluids in the liquid-vapour equilibrium has such an alternative and we assume that this alternative is also available to the population of hawks and doves in phase equilibrium between A and B. The alternative is that, – in the language of populations rather than solutions –, the population becomes non-homogeneous, *i.e.* it falls apart into colonies of different hawk fractions.

The reason for the decomposition into colonies is best explained by referring to Fig. 2. That figure shows the graphs of $F^A(x)$ and $F^B(x)$ for V=2.70 which intersect twice. The straight dashed lines in the figure are tangent to $F^A(x)$ and they connect the points of contact of the tangents with the end-points of $F^B(x)$ at x=0 and x=1. Along these tangents the energy of a non-homogeneous population is lower than the energy of the homogeneous cohabitation with the overall fraction x. Therefore colonies form for those values V for which tangents of the type shown in Fig. 2a occur, – with compositions appropriate to the end-points of the tangents. At least that is the assumption; it is motivated by what is indeed observed in solutions and alloys.***

In the case where the overall composition x is under the left tangent of Fig. 2a the colonies are pure dove and a hawk-dove cohabitation of phase A with hawk fraction x_C^1. The fraction of doves assembled in pure dove colonies is $\frac{x_C^1-x}{x_C^1}$; obviously it depends on V.

In the case where the overall composition x lies below the right tangent of Fig. 2a the colonies are pure hawk and a hawk-dove cohabitation of phase A with the hawk fraction x_C^2. The fraction of hawks assembled in pure hawk colonies is $\frac{x-x_C^2}{1-x_C^2}$.

If the overall x does not lie under the tangents, we have no colonies but rather a homogeneous population of hawks and doves in cohabitation in phase A.

But this is not all. For higher values of V the population becomes non-homogeneous in yet another manner, illustrated by Fig. 2b. In that case along the dashed line connecting the end-points of $F^B(x)$ the energy is smaller than in any point of

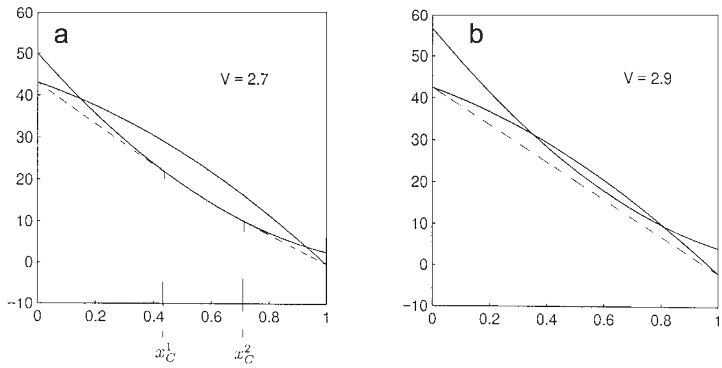

Fig. 2 Energies $F^A(x)$ and $F^B(x)$ and the dashed lines equilibrium between colonies. a – V=2.70. Colonies consist of pure hawk, or pure dove and hawk-dove mixture with a hawk fraction x_C. b – V=2.90. Colonies consist of pure hawk and pure dove

*** See any book on chemical thermodynamics, *e.g.* [4]

$F^{A}(x)$ or $F^{B}(x)$. Therefore colonies form of pure hawk and pure dove. At least they would, if the population follows the same rules as solutions and alloys do. The fraction of hawks assembled in pure hawk colonies obviously equals x in that case.

Phase diagram

It is obvious from the construction of the dashed lines in Fig. 2 that the values x_C depend on V. In fact, since the energies $F^{A}(x)$ and $F^{B}(x)$ are parabolic, it is easy to calculate the points of contact. Table 1 presents the results for some values V in the interesting range.

Table 1 Left and right contact points of tangents (Fig. 2a) for different values V

V	x_C^1	x_C^2
2.48996	–	1
2.49605	0	0.949
2.5	0.066	0.934
2.6	0.341	0.777
2.7	0.481	0.686
2.8	0.593	0.611

In Fig. 3 we use the values of this table to construct the corresponding phase diagram V *vs*. x. The two graphs $x_C^1(V)$ and $x_C^2(V)$ intersect each other at a value $V \gtrsim 2.8$. In terms of Fig. 2a this means that the two tangents are becoming one. The point of intersection defines the end of phase A as V goes up. The phase diagram also shows the various areas of segregation and cohabitation as discussed before. We have unrestricted cohabitation at small values V in phase A and complete segregation into hawks and doves for large values of V when phase B is energetically favourable. We note that phase B does not allow cohabitation. For intermediate values of V we have

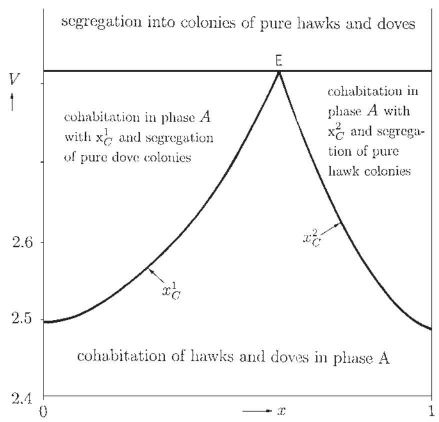

Fig. 3 Phase diagram with areas of cohabitation and segregation

cohabitation in phase A but with colonies of hawks or doves segregated off to a varying degree.

The highest point where cohabitation is possible is marked by a dot in Fig. 3 and denoted by E. In analogy to solutions and alloys this point is called the eutectic point. For higher values of V no cohabitation of hawks and doves is possible; the two characters split into colonies of pure hawk and pure dove. In the language of metallurgy or physical chemistry this is the miscibility gap.

Discussion

For those readers familiar with metallurgy or chemical engineering the situation described by Fig. 3 will be perfectly clear. To be sure, the variables are different but the phenomena are alike. What is V here, the value of the resource, is the pressure for a liquid solution in equilibrium with its vapour phase. And the hawk fraction of our case corresponds to the concentration, or mol-fraction, of the solution. The low-V-phase A is the vapour phase in chemical engineering and the high-V-phase B is the liquid solution with completely immiscible components.

Still, not everybody may be familiar with thermodynamics of solutions and for those who are not we describe what happens to a population of a fixed hawk fraction x^0 as the value V goes up, starting from a point in the low-V-, or A-phase, (Fig. 4a).

At first nothing much occurs; the phase A remains stable until the segregation line, denoted by x_C^1, is reached. When that is the case, a small colony of pure doves will form making the bulk of phase A a little richer in hawks, so that V can be raised a little higher, before the segregation line is again reached with the same result as before: another pure-dove-colony forms and the hawk ration increases in the remaining phase A. This goes on; the state of the pure dove colonies creeps up on the V-axis and the state of the remaining phase A with hawks and doves in cohabitation creeps up on the segregation line, as indicated by the arrows. For each value V the fraction of doves

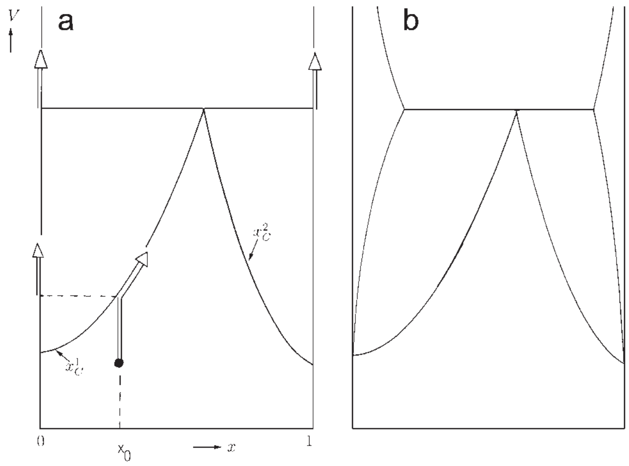

Fig. 4 a – Increasing V; b – Phase diagram with colonies of limited cohabitation

in colonies may be determined by the ratio of the lengths $x_C^1 - x^0$ and x_C^1. This goes on until the eutectic point is reached by the remaining phase A. When V is raised further, colonies of pure hawks and pure doves form from the remaining phase A in the proportion $x_E/(1-x_E)$, where x_E is the abscissa of the eutectic point. The complete segregation of hawks and doves persists for all higher values of V, each group occupying their colonies. Each colony moves up vertically, the doves at x=0 and the hawks at x=1, as indicated by the arrows in Fig. 4.

Outlook

It is true that in the physical world of solutions and alloys it is more usual to encounter phase diagrams as the one in Fig. 4b than the one in Fig. 4a. This means that there is always some miscibility, however little. This is to be expected in populations as well; there should not be pure hawk or pure dove colonies. The fact that we have obtained such pure colonies is due to our neglect of an entropic term in the energy. Such a term has the form

$$kT(x\ln x+(1-x)\ln(1-x)) \quad (7)$$

where T, the temperature, must be defined as a measure for the ‘thermal’ motion, or the random migration of hawks and doves in the population. We have no convincing idea – at this time – for a quantitative value of T and therefore we have neglected the entropic term altogether. Clearly this subject offers the opportunity for further study.

Another prospect is the incorporation of a rate of change of the overall hawk fraction x due to evolutionary processes. In the language of chemical thermodynamics this means to take chemical reactions into account.

The research presented here will continue with the objective of transferring the well-known tenets of the theory of mixtures, solutions and alloys into sociology and sociobiology. It is hoped that the successful conclusion of this effort will shed a new light on the ‘mechanisms’ of sociobiological phenomena.

Finally we like to say that what has been argued here for a synthetic population of hawks and doves may well be extrapolated by a courageous sociologist to circumstances closer at home, *e.g.* integration versus segregation of human groups of different religious or ethnic backgrounds. Mimkes [5] is pursuing a similar, albeit more phenomenological path with stimulating and thought-provoking results.

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